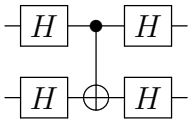
Problem Set 4 - LV 141.A55 QISS - 18.4.2016

1. **Controlled Not in a Different Bases**

   Show that Hadamard gate performs a transformation from the z to the x-axes.

   What happens with a y-state $|y\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)$?

   Calculate the controlled not gate in the x bases?



2. **Universal Two Qubit Gate**

$$\sqrt{\text{SWAP}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   Show that the gate $\sqrt{\text{SWAP}}$ is a universal two-qubit gate by building a CNOT out of one or several $\sqrt{\text{SWAP}}$ and single qubit operations.

   Hint: one needs two $\sqrt{\text{SWAP}}$.

   The SWAP gate is not a universal. Can you find an argument?

3. **Two Qubit Interactions**

   Consider the following two qubit interaction Hamiltonians and study unitary evolution. For how long do you have to apply these interactions in order to create a universal gate.

   (a) Ising interaction

$$H_{zz} = -j_{zz}\sigma_{z,1}\sigma_{z,2}$$

   (b) Heisenberg interaction

$$H_{jj} = -j_{jj}(\sigma_{x,1}\sigma_{x,2} + \sigma_{y,1}\sigma_{y,2} + \sigma_{z,1}\sigma_{z,2})$$

   (c) XY interaction

$$H_{xy} = -j_{xy}(\sigma_{x,1}\sigma_{x,2} + \sigma_{y,1}\sigma_{y,2})$$